

IC/99/10

# New Formulas and Predictions for Running Masses at Higher Scales in MSSM

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## Abstract

Including contributions of scale-dependent vacuum expectation values of Higgs scalars ,we derive new one-loop formulas analytically for running quark-lepton masses at higher scales in the MSSM .Apart from the gauge-coupling dependence of all masses being different from earlier formulas,the third-generation-Yukawa-coupling effects are absent in the masses of the first two generations. While predicting the masses and  $\tan \beta$ ,numerically, we also include two-loop effects.

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One of the important objectives of current researches in High Energy Physics is to understand the masses and mixings of quarks and leptons in the context of a unified theory of basic interactions. Apart from accounting for the well known gauge hierarchy problem, the minimal supersymmetric standard model (MSSM) remarkably exhibits the unification of gauge couplings of the standard model (SM) at  $M_U \simeq 2 \times 10^{16} \text{ GeV}$  consistent with the CERN-LEP data [1]. The knowledge of running particle masses is not only essential near the electro-weak scale, but also near the intermediate and GUT scales in order to test successes of models based upon quark-lepton unification [2], infrared structure of Yukawa couplings [3], Yukawa textures for fermion masses [4], and predictive ansatz for fermion masses and mixings in  $SO(10)$  [5]. For model building, including flavour symmetry to explain neutrino masses and mixings, it might be necessary to know, how the masses of the first two generations behave with respect to the third generation Yukawa couplings. The possibility that quark-lepton unification might exist at the GUT scale was strongly indicated [6] where the effects of gauge couplings on running masses were derived analytically and the necessity of analytic formulas including Yukawa coupling effects has been emphasized [7]. While the top-down approach predicts the particle masses and mixings in terms of GUT-scale parameters, the bottom-up approach predicts the running masses at higher scales in terms of experimentally determined values at low energies. In particular, the quark-lepton masses at higher scales ( $\mu > m_t$ ) are predicted through one-loop formulas [8],

$$\begin{aligned}
m_t(\mu) &= m_t(m_t) A_u^{-1} e^{(6I_t + I_b)} \\
m_c(\mu) &= m_c(m_c) \eta_c^{-1} A_u^{-1} e^{3I_t}, m_u(\mu) = m_u(1 \text{ GeV}) \eta_u^{-1} A_u^{-1} e^{3I_t} \\
m_b(\mu) &= m_b(m_b) \eta_b^{-1} A_d^{-1} e^{(I_t + 6I_b + I_\tau)} \\
m_i(\mu) &= m_i(1 \text{ GeV}) \eta_i^{-1} A_d^{-1} e^{(3I_b + I_\tau)}, i = s, d \\
m_\tau(\mu) &= m_\tau(m_\tau) \eta_\tau^{-1} A_e^{-1} e^{(3I_b + I_\tau)} \\
m_i(\mu) &= m_i(m_i) \eta_i^{-1} A_e^{-1} e^{(3I_b + I_\tau)}, i = \mu, e
\end{aligned} \tag{1}$$

where the Yukawa-coupling ( $y_f$ ) integrals are defined as,  $I_f = (1/16\pi^2) \int_{\ln m_t}^{\ln \mu} y_f^2(t) dt$ ,  $f = t, b, \tau$ ; and

$$\begin{aligned}
A_u &= (\alpha_1(\mu)/\alpha_1(m_t))^{13/198} (\alpha_2(\mu)/\alpha_2(m_t))^{3/2} (\alpha_3(\mu)/\alpha_3(m_t))^{-8/9} \\
A_d &= (\alpha_1(\mu)/\alpha_1(m_t))^{7/198} (\alpha_2(\mu)/\alpha_2(m_t))^{3/2} (\alpha_3(\mu)/\alpha_3(m_t))^{-8/9} \\
A_e &= (\alpha_1(\mu)/\alpha_1(m_t))^{3/22} (\alpha_2(\mu)/\alpha_2(m_t))^{3/2}
\end{aligned} \tag{2}$$

All running-masses occurring on the R.H.S. of eq.(1) are determined from experimental measurements using their respective relations to the pole masses. The parameters  $\eta_\alpha$  ( $\alpha = u, c, d, s, b, e, \mu, e$ ) are the QCD-QED rescaling factors [9, 10, 11]. Assuming SUSY breaking scale at  $\mu = m_t$ , these formulas are derived using analytic solutions to one-loop RGEs of Yukawa-coupling eigenvalues [8, 9, 10, 11] and the relation among mass matrix  $M_a$ , the Yukawa matrix  $Y_a$ , and the VEV  $v_a$ ,

$$M_a(\mu) = Y_a(\mu)v_a \quad (3)$$

where  $v_a = v_u = v_0 \sin \beta$ ,  $a = U$  and  $v_a = v_d = v_0 \cos \beta$ ,  $a = D, E$ , with  $v_0 = 174.0 \text{ GeV}$ . The running masses of all fermions of the first two generations are seen to depend upon third-generation-Yukawa-coupling integrals. In deriving these formulas, the only scale dependence that has been assumed is through the Yukawa couplings,  $Y_a(\mu)$ , and not through the vacuum expectation values (VEVs),  $v_a$ , of eq.(4). Numerical values of masses have also been reported very recently totally neglecting the scale dependence of the VEVs [10]. Below the electroweak symmetry breaking scale, the VEVs do not depend upon the mass scales ( $\mu$ ) and the running masses, related to their corresponding pole masses, are to be taken as the actual ansatz of the MSSM when the  $\mu$ -dependence of VEVs are ignored. But, above the electroweak-symmetry breaking scale, the  $\mu$ -dependence of the VEVs are well defined [12, 13], through their RGEs and beta functions upto two-loops,

$$\begin{aligned} 16\pi^2(d \ln v_{u,d}/dt) &= \beta_{v_{u,d}} + \text{two-loops} \\ \beta_{v_u} &= 3g_1^2/20 + 3g_2^2/4 - 3\text{Tr}(Y_U Y_U^\dagger) \\ \beta_{v_d} &= 3g_1^2/20 + 3g_2^2/4 - \text{Tr}(3Y_D Y_D^\dagger + Y_E Y_E^\dagger) \end{aligned} \quad (4)$$

The  $\mu$ -dependence of the VEV in the SM has been considered in the context of infrared fixed points [3] and, very recently, to predict CKM parameters and the top-quark mass at higher scales [14]. Here we confine ourselves to the case of MSSM only. The RGEs for the mass matrices for  $\mu > m_t$  are obtained in a straight-forward manner combining the corresponding RGEs for  $Y_a$  and  $v_a$ ,

$$\begin{aligned} 16\pi^2(dM_U/dt) &= (-c_i g_i^2 + 3Y_U Y_U^\dagger + Y_D Y_D^\dagger) M_U \\ 16\pi^2(dM_D/dt) &= (-c'_i g_i^2 + Y_U Y_U^\dagger + 3Y_D Y_D^\dagger) M_D \\ 16\pi^2(dM_E/dt) &= (-c''_i g_i^2 + Y_E Y_E^\dagger) M_E \end{aligned} \quad (5)$$

where  $c_i = (43/60, 9/4, 16/3)$ ,  $c'_i = (19/60, 9/4, 16/3)$ , and  $c''_i = (33/20, 9/4, 0)$ . Defining the diagonal mass matrices( $\hat{M}_F$ ) and Yukawa matrices( $\hat{Y}_F$ ) through biunitary transformation and the CKM matrix( $V$ ) as [9],  $\hat{M}_F = L_F^\dagger M_F R_F$ ,  $V = L_U^\dagger L_D$ ,  $\hat{M}_F^2 = L_F^\dagger M_F M_F^\dagger L_F$ ,  $\hat{Y}_F^2 = L_F^\dagger Y_F Y_F^\dagger L_F$ , we derive RGEs for  $\hat{M}_F^2$ ,

$$\begin{aligned} d\hat{M}_U^2/dt &= [\hat{M}_U^2, L_U^\dagger \dot{L}_U] + (1/16\pi^2)(-2c_i g_i^2 \hat{M}_U^2 + 6\hat{Y}_U^2 \hat{M}_U^2 + V\hat{Y}_D^2 V^\dagger \hat{M}_U^2 + \hat{M}_U^2 V\hat{Y}_D^2 V^\dagger) \\ d\hat{M}_D^2/dt &= [\hat{M}_D^2, L_D^\dagger \dot{L}_D] + (1/16\pi^2)(-2c'_i g_i^2 \hat{M}_D^2 + 6\hat{Y}_D^2 \hat{M}_D^2 + V^\dagger \hat{Y}_U^2 V \hat{M}_D^2 + \hat{M}_D^2 V^\dagger \hat{Y}_U^2 V) \\ d\hat{M}_E^2/dt &= [\hat{M}_E^2, L_E^\dagger \dot{L}_E] + (1/16\pi^2)(-2c''_i g_i^2 \hat{M}_E^2 + 6\hat{Y}_E^2 \hat{M}_E^2) \end{aligned} \quad (6)$$

where the dot inside the commutator on the RHS denotes the derivative with respect to the variable  $t = \ln \mu$ . The diagonal elements of  $L_F^\dagger \dot{L}_F$ , ( $F = U, D, E$ ) are fixed to be zero in the usual manner [9] through diagonal phase multiplication. The RGEs for the Yukawa and the CKM matrix elements remain the same as before [8, 9, 15]. Now using the diagonal elements of both sides of eqs.(6), the RGEs for the mass eigen values are obtained by ignoring the Yukawa couplings of first two generations,

$$\begin{aligned} 16\pi^2(dm_j/dt) &= [-c_i g_i^2 + |V_{jb}|^2 y_b^2] m_j, j = u, c \\ 16\pi^2(dm_t/dt) &= [-c_i g_i^2 + 3y_t^2 + |V_{tb}|^2 y_b^2] m_t \\ 16\pi^2(dm_j/dt) &= [-c'_i g_i^2 + |V_{tj}|^2 y_t^2] m_j, j = d, s \\ 16\pi^2(dm_b/dt) &= [-c'_i g_i^2 + 3y_b^2 + |V_{tb}|^2 y_t^2] m_b \\ 16\pi^2(dm_j/dt) &= [-c''_i g_i^2] m_j, j = e, \mu \\ 16\pi^2(dm_\tau/dt) &= [-c''_i g_i^2 + 3y_\tau^2] m_\tau \end{aligned} \quad (7)$$

Integrating these equations and using the corresponding low-energy values, the new formulas are obtained in the small mixing limit as,

$$\begin{aligned} m_t(\mu) &= m_t(m_t) B_u^{-1} e^{(3I_t + I_b)} \\ m_c(\mu) &= m_c(m_c) \eta_c^{-1} B_u^{-1} \\ m_u(\mu) &= m_u(1\text{GeV}) \eta_u^{-1} B_u^{-1} \\ m_b(\mu) &= m_b(m_b) \eta_b^{-1} B_d^{-1} e^{(I_t + 3I_b)} \\ m_i(\mu) &= m_i(1\text{GeV}) \eta_i^{-1} B_d^{-1}, i = d, s \\ m_\tau(\mu) &= m_\tau(m_\tau) \eta_\tau^{-1} B_e^{-1} e^{3I_\tau} \\ m_i(\mu) &= m_i(m_i) \eta_i^{-1} B_e^{-1}, i = e, \mu \end{aligned} \quad (8)$$

where

$$\begin{aligned}
B_u &= (\alpha_1(\mu)/\alpha_1(m_t))^{43/792}(\alpha_2(\mu)/\alpha_2(m_t))^{9/8}(\alpha_3(\mu)/\alpha_3(m_t))^{-8/9} \\
B_d &= (\alpha_1(\mu)/\alpha_1(m_t))^{19/792}(\alpha_2(\mu)/\alpha_2(m_t))^{9/8}(\alpha_3(\mu)/\alpha_3(m_t))^{-8/9} \\
B_e &= (\alpha_1(\mu)/\alpha_1(m_t))^{1/8}(\alpha_2(\mu)/\alpha_2(m_t))^{9/8}
\end{aligned} \tag{9}$$

For  $\tan \beta = v_u/v_d$ , the RGE is obtained from the difference of the beta functions,  $\beta_{v_u} - \beta_{v_d}$ , and the values at higher scales are given by the one-loop analytic formula,

$$\tan \beta(\mu) = \tan \beta(m_t) e^{(-3I_t + I_b + I_\tau)} \tag{10}$$

It is clear from (8)-(9) that, compared with (1)-(2), the new formulas have very significant differences with respect to their functional dependence on gauge and Yukawa couplings in all cases. Also the masses of the first two generations are found to be independent of third generation Yukawa couplings. While deriving one-loop formulas using the see-saw mechanism in SUSY GUTs, it has been shown that the left-handed neutrino masses of the first two generations have no additional dependence on Yukawa couplings except through respective up-quark masses [16]. Combining the present result with that of ref. [16], it turns out that all fermion masses of first two generations of MSSM are independent of Yukawa couplings of the third generation. In view of the present results, apart from modifying analytic formulas, earlier numerical mass predictions including ref. [10], where  $\mu$ -dependence of VEVs have been ignored, are to be rescaled by  $v_u(\mu)/v_0$  for up-quark masses and by  $v_d(\mu)/v_0$  for down-quark and charged lepton masses. While estimating masses, VEVs, and  $\tan \beta$  at higher scales, we have solved all relevant RGEs, including those of VEVs and  $\tan \beta(\mu)$ , upto two-loops with the same inputs at  $\mu = m_t$  as in ref. [10]. We find that the input value of  $m_t(m_t) = 171 \pm 12$  GeV gives rise to the perturbative limit  $y_t(M_{GUT}) \leq 3.54$  at  $\tan \beta(m_t) \geq 1.74_{-28}^{+46}$ . Due to running governed by the corresponding RGE at two-loop level, this limit at the GUT-scale turns out to be  $\tan \beta(M_{GUT}) \geq .52_{-10}^{+14}$ . To our knowledge, this is the first result in the literature, showing that actual solutions to RGEs permit  $\tan \beta(M_{GUT}) (\equiv \tan \bar{\beta}) < 1$  near the perturbative limit of  $y_t(M_{GUT})$ . This gives rise to the possibility of a perturbation expansion in terms of  $\tan \bar{\beta}$  in this region. We also observe the saturation of the perturbative limit for the b-quark Yukawa coupling ( $\bar{y}_b$ ) for

$\tan\beta(m_t) \simeq 61$ , similar to [10]. We have checked that the one loop analytic solutions agree with the full two-loop numerical solutions within 5-7% of accuracy except near the perturbative limits, where the discrepancy increases further due to larger two-loop effects.

In Table I, we present the predictions for VEVs,  $\tan\beta$  and masses at two different scales:  $\mu = 10^9 \text{ GeV}$ , and  $\mu = 2 \times 10^{16} \text{ GeV}$  for the input  $\tan\beta(m_t) = 10$ . Our solutions of RGEs yield very significantly different values of  $v_u(\mu)$  than the assumed scale independent one, although  $v_d(\mu)$  is not very significantly different, for such a  $\tan\beta \approx 10$ . This feature leads to quite different up-quark masses, the most prominent being  $m_t(\mu)$ . The running VEV of  $v_u$  reduces the central value of  $m_t(\mu)$  to nearly 72%(57%) at the intermediate(GUT) scale. Similarly,  $m_u(\mu)$  and  $m_c(\mu)$  are reduced to 80%(67%) and 79%(66%), respectively, at the intermediate(GUT)-scale as compared to [10]. As  $v_d(\mu)$  is closer to the assumed scale-independent value for  $\tan\beta \simeq 10$ , all the down quark and the charged lepton masses are closer to the values obtained in ref. [10]. But, it is clear, that significant differences will appear in these cases also from the computations based upon scale-independent assumption, in the larger  $\tan\beta$ -region.

In Table II, we present GUT-scale predictions of VEVs,  $\tan\beta$  and third generation fermion masses, denoted with overbars, as a function of different input values of  $\tan\beta(m_t)$ . The GUT-scale value of  $m_t(M_{GUT})$  is found to reach nearly a minimum, which is approximately half of its perturbative-limiting value, for  $\tan\beta \simeq 10$ . After this minimum is reached,  $m_t(M_{GUT})$  increases slowly with increasing  $\tan\beta$ ; but the increase is faster for  $\tan\beta \geq 50$ . Similarly  $m_b(M_{GUT})$  shows more than 10% increase both for smaller(larger) values of  $\tan\beta$  below(above) 2.0(40.0) as compared to its value at  $\tan\beta = 10$ . Also the numerical solution to RGE for  $\tan\beta(\mu)$  exhibits its GUT-scale value ( $\tan\bar{\beta}$ ) to be significantly less than the low energy input except until the input approaches the value of  $\tan\beta(m_t) \simeq 61$  corresponding to the saturation of the perturbative limit of  $y_b(M_{GUT})$ . In this region, the GUT-scale value of  $\tan\bar{\beta}$  exceeds the corresponding low energy input as shown for the case of  $\tan\beta(m_t) = 60$ . We conclude that inclusion of the effects of running VEVs yields completely new fermion mass formulas with respect to their dependence on gauge and Yukawa couplings. In view of the results presented here and formulas for neutrino masses [16], all running masses of first two generations in the MSSM are independent of runnings of third-generation- Yukawa couplings. This behavior of masses may be contrasted with that of Yukawa

couplings ,which, for the first two-generations,depend upon the couplings of the third.Numerical estimations yield very significantly different values on masses at higher scales and provides interesting new informations on the GUT-scale values of  $\tan \beta$ .

One of us(M.K.P.) thanks Professor Goran Senjanovic and Professor K.S.Babu for useful discussions and Professor S.Randjbar-Daemi for encouragement.The AS ICTP Associateship grants of M.K.P are gratefully acknowledged.

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Table 1: **Predictions of masses ,VEVs, and  $\tan\beta(\mu)$  at two different scales in MSSM for  $\tan\beta(m_t) = 10.0$  and other low-energy values same as in ref. [10].**

Parameter	This analysis $\mu = 10^9 GeV$	Ref.[10]	This analysis $\mu = 2 \times 10^{16} GeV$	Ref.[10]
$\tan\beta$	7.973	10	6.912	10
$v_u(GeV)$	142.123	173.130	128.085	173.130
$v_d(GeV)$	17.815	17.312	18.534	17.312
$m_t(GeV)$	107.52	$149^{+40}_{-26}$	73.55	$129^{+96}_{-40}$
$m_c(GeV)$	.3373	$.427^{+.035}_{-.038}$	.2003	$.302^{+.025}_{-.027}$
$m_u(MeV)$	1.178	$1.470^{+.26}_{-.28}$	.7059	$1.04^{+.19}_{-.20}$
$m_b(GeV)$	1.580	$1.60 \pm .06$	1.004	$1.00 \pm .04$
$m_s(GeV)$	.0478	$.0453^{+.0057}_{-.0063}$	.0292	$.0265^{+.0033}_{-.0037}$
$m_d(MeV)$	2.4018	$2.28^{+.29}_{-.32}$	1.4632	$1.33^{+.17}_{-.19}$
$m_\tau(GeV)$	1.5177	$1.4695^{+.0003}_{-.0002}$	1.2566	$1.1714 \pm .0002$
$m_\mu(MeV)$	89.088	$86.217 \pm .00028$	73.6226	$68.59813 \pm .00022$
$m_e(MeV)$	.422	.40850306	.3487	.32502032

Table 2: **Predictions of VEVs, $\tan\beta$ ,and third generation fermion masses at the GUT-scale as a function of input values of  $\tan\beta(m_t)$  and other input masses same as in ref.[10].The GUT-scale values have been denoted with overbars.**

$\tan\beta(m_t)$	$\tan\beta$	$\bar{v}_u$	$\bar{v}_d$	$\bar{m}_t$	$\bar{m}_b$	$\bar{m}_\tau$
1.75	.521	48.497	93.078	146.144	1.280	1.253
2	.963	80.87	83.90	100.962	1.116	1.252
5	3.35	123.13	36.74	75.138	1.008	1.252
10	6.910	128.08	18.54	73.556	1.004	1.256
20	14.18	128.75	9.079	73.92	1.022	1.2739
30	22.1	127.899	5.787	75.321	1.0613	1.3078
40	31.443	126.15	4.0119	77.869	1.1317	1.3662
50	44.476	123.05	2.766	82.768	1.274	1.484
60	80.60	116.08	1.440	98.103	1.820	1.924